MODELING OF SOLAR-RADIATION TRANSFER IN THE SPHERICAL SYSTEM OF ATMOSPHERE–EARTH'S SURFACE

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The study gives a characteristic of the developed model of solar-radiation transfer in the spherical system atmosphere–earth's surface. Account is taken of the vertical profiles of the aerosol and gas components of the atmosphere, the anisotropy of the reflection of the earth's mantle, the properties of the relief of the earth's surface, the characteristics of the wind wave, and the optical properties of the aqueous medium. The problem of radiation transfer is considered in the approximation of the small-angle modification of the method of spherical harmonics. The polar and azimuthal distributions of the components of the Stokes vector are obtained. The integral radiation characteristics and the temperature and rate of heating of the medium at various levels are determined within the framework of a general calculation scheme.

Introduction. The field of solar radiation is one of the determining components of the earth's ecosystem and biosphere. It affects the mechanisms of the variability of the geophysical, meteorological, and climatic state of the earth [1]. These circumstances necessitate studying and predicting, based on mathematical modeling of radiation transfer, the natural processes and mechanisms of formation of anomalous states of the natural environment under disturbing effects such as the consequences of technogenic accidents, natural catastrophes, anthropogenic loading, etc. In this connection, of great importance are investigations aimed at working out remote methods of monitoring natural and anthropogenic disturbances of the environment [2].

The actual pattern of radiation processes in the earth–atmosphere system is fairly intricate. It depends on a number of thermodynamic parameters, a great many scattering and absorbing substances, the reflective and radiative properties of the underlying surface, and the specific features of their regional distribution. Existing mathematical models of large-scale processes in the atmosphere [3-5] are unable to adequately reproduce this multiparameter pattern in its entirely.

Physicomathematical Model. The propagation of solar radiation in the spherical system of atmosphere–earth's surface is described by a boundary-value problem of transfer theory whose solution is the radiation intensity (radiance) $I(\mathbf{r}, \mathbf{s})$. To solve the equation of radiation transfer in the earth's atmosphere we selected a spherical system of coordinates with the axis positioned in the direction of the local zenith. In this system of coordinates, the position of the point $\mathbf{r}$ is determined by its distance from the planet center r and the angle ψ between the direct sunbeam and the radius vector drawn to this point. The direction of radiation $\mathbf{s}$ at the point is specified by the zenithal distance ϑ and the azimuth φ measured from the plane of the sun's vertical. Because the problem is symmetric, in sunlight the radiation intensity in this system of coordinates is a function of four variables $I = I(r, \psi, \vartheta, \varphi)$, and the equation of radiation transfer for the steady-state problem is written as

$$\cos\vartheta \frac{\partial I}{\partial r} - \sin\vartheta \frac{\partial I}{r\partial\vartheta} + \sin\vartheta\cos\varphi \frac{\partial I}{r\partial\psi} -$$

$$- \sin\vartheta \sin\varphi \operatorname{ctan}\psi \frac{\partial I}{r\partial\varphi} + \varepsilon I - \frac{\varepsilon\Lambda}{4\pi} \int_0^{2\pi} \int_0^{\pi} I' x(\theta) \sin\vartheta' d\vartheta' d\varphi' =$$

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$$= \frac{\varepsilon\Lambda}{4\pi} x(\theta_0) \pi F_0 \exp\left\{-\tau(r, \psi)\right\} + \varepsilon(1-\Lambda) B(r, \psi). \tag{1}$$

The first term on the right-hand side of Eq. (1) defines the distribution of the sources of single scattering for shortwave solar radiation, and the second, the distribution of the sources of longwave self-radiation. Here, the attenuation factor of the radiation in the medium $\varepsilon = \sigma + \alpha$ is expressed in terms of appropriate scattering and absorption indices; $\Lambda = \sigma/\varepsilon$ is the probability of quantum survival (the albedo of single scattering); the optical thickness $\tau(r, \psi)$ is represented by an integral expression of the attenuation factor over the path of the direct beam; the primed quantities relate to radiation incident on an elementary volume of the medium.

The optical properties of the atmosphere are specified by the height profiles $\varepsilon(r)$ and $\Lambda(r)$ and the scattering indicatrix of an elementary volume $x(r, \theta)$ represented as the weighted-average function of the scattering angle

$$x(r, \theta) = q_{\mathrm{m}}(r)\, x_{\mathrm{m}}(\theta) + q_{\mathrm{a}}(r)\, x_{\mathrm{a}}(\theta) \tag{2}$$

with the aid of weight functions $q_{\mathrm{m}} = \sigma_{\mathrm{m}}/\sigma$ and $q_{\mathrm{a}} = \sigma_{\mathrm{a}}/\sigma$ that express the relative role of the molecular and aerosol components [6]. Different stages of transformation of the aerosol of the lower terrestrial atmosphere are modeled by varying the relative volume concentration (content) of the main polydisperse aerosol components: D, dust-like water-insoluble particles of soil origin; W, water-soluble particles such as ammonia, calcium sulfate, and organic compounds; S, soot anthropogenic aerosol; O, ocean aerosol [7].

The principle of adding the scattering and absorption indices is used to form layers of atmospheric and oceanic stratification. Thus the optical properties of the tropospheric aerosol layer, dust and smoke layers, fog, cloudiness of various levels, the stratospheric aerosol layer, the ozone layer, and other gas components are reproduced, as are those of the phyto- and zooplankton layers in the aqueous medium.

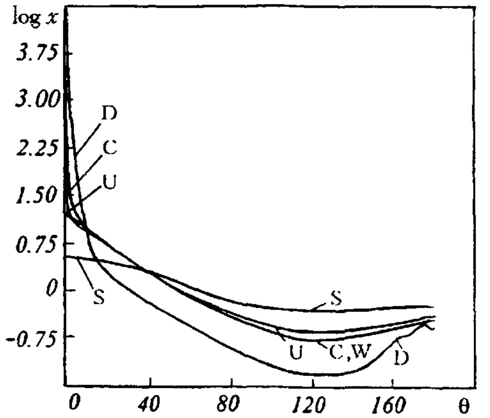
The boundary conditions for Eq. (1) express the absence of diffuse radiation incident on the system from outside. Passage of radiation through the phase (air–water) boundary is described by means of the reflection and transmission operators [8].

The equation of radiation transfer is solved using one of the efficient methods, namely, the method of spherical harmonics, which in principle gives results with any degree of accuracy [9]. The most important optical parameter that determines the degree of complication of calculations by this method is the scattering indicatrix $x(\theta)$. Actual $x(\theta)$ have a marked forward peak due to the presence of large scattering particles and require account for tens and hundreds of terms in a series expansion in Legendre polynomials $P_L^{00}(\mu)$, $\mu = \cos\theta$. An approximate solution (a P_n-approximation) of transfer equation (1) is sought in the form

$$I(\mathrm{r}, \mathrm{s}) = \sum_{L=0}^{n} (2L+1) \sum_{m=-L}^{L} I_L^m(\mathrm{r})\, Y_L^m(\mathrm{s}), \tag{3}$$

where $I_L^{\mathrm{m}}(\mathrm{r})$ are unknown functions of the spatial coordinates, and $Y_L^{\mathrm{m}}(\mathrm{s})$ are spherical functions. The boundary-value problem reduces to solving a system of $(n+1)^2$ partial differential equations in the functions $I_L^{\mathrm{m}}(\mathrm{r})$.

The solution of scalar transfer equation (1) for the intensity is incapable of reflecting all structural features of scattered solar radiation. The information capacity of the energy characteristics of scattered radiation is limited significantly. Practically the entire measurable information on the optical constants of the substance, the microstructure and component composition of the aerosol, and the properties of the water surface and the earth's mantle is contained in the polarization characteristics of scattered radiation. Their use is very promising in problems of remote optical monitoring of natural and anthropogenic disturbances in the system of atmosphere–earth's surface–ocean. In this case, the solution of the boundary-value problem for the equation of solar-radiation transfer is a vector analog of intensity, namely, the Stokes vector containing four components $\mathbf{I} = \{I, Q, U, V\}$. The light scattering indicatrix is replaced by a 4×4 matrix. In the case of spherical symmetry, which often corresponds to the aerosol, the scattering matrix contains eight nonzero elements:



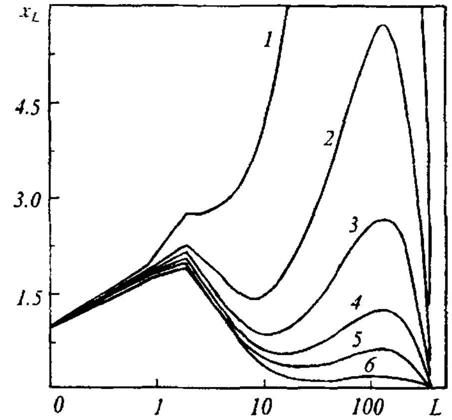


Fig. 1. Light scattering indicatrices for models of the surface (continental C and urban U) aerosol and its components D, W, and S.

Fig. 2. Spectra of expansion coefficient for diagonal elements of a scattering matrix. The content of the dust component $C_D = 0.9$ (1), 0.7 (2 = C), 0.5 (3), 0.3 (4), 0.17 (5 = U), 0.05 (6).

$$\| x_{ij}(\theta) \| = \begin{bmatrix} x_{11} & x_{12} & 0 & 0 \\ x_{21} & x_{22} & 0 & 0 \\ 0 & 0 & x_{33} & x_{34} \\ 0 & 0 & x_{43} & x_{44} \end{bmatrix} . \tag{4}$$

In solving the problem of radiation transfer by the method of spherical harmonics, the elements of the scattering matrix are expanded in a series in generalized spherical functions $P_L^{mn}(\mu)$. Distributions (discrete spectra) of the expansion factors are calculated by the equation

$$x_L^{ij} = \frac{2L+1}{2} \int_{-1}^{1} x_{ij}(\mu)\, P_L^{mn}(\mu)\, d\mu , \tag{5}$$

where the index L is the number of the expansion factor.

Results of Numerical Modeling. Figure 1 presents light scattering indicatrices calculated from the Mie equations for models of the surface aerosol. The content C_k of the components $k = \{D, W, S\}$ in the models is 0.70, 0.29, and 0.01 for the continental aerosol C and 0.17, 0.61, and 0.22 for the urban aerosol U, respectively. For the models of the aerosols C and U, and the component D, the indicatrices exhibit a pronounced asymmetry with a sharp peak in the region of small scattering angles. A significant portion of the energy of scattered radiation is concentrated here. As is seen from the figure, the finely disperse components S and W have indicatrices differing slightly from the Rayleigh indicatrix. The angular functions $x(\vartheta)$ calculated for the models of the urban U and continental C aerosol are similar in form and weakly exhibit a variation in the input parameters.

The calculated data for the spectra presented in Fig. 2 clearly reflect the presence, in the aerosol mixture, of particles of fine (the first maximum) and coarse (the second maximum) fractions and the dynamics of the variation in the parameters of the microstructure and the component composition of the aerosol during its transformation. Here, the positions of the maxima L_k are linearly related to the effective radii of the particles of the fractions, $r_{ef}^k = c\lambda L_k$, $\lambda = 0.55\ \mu$m is the wavelength of the light, and $c = 0.11-0.14$. Here, $k = \{1, 2\}$ are the numbers of the fractions.

The results of statistical modeling of the spectra with allowance for possible errors in the input parameters (a relative error $\delta x = 0.1$ and absolute errors $\Delta C_{D,W} = 0.05$) reveal that the amplitude of the spectra in the second maximum bears a close stochastic relation (with a correlation factor of 0.5–0.9) to the content C_D of the dust component and the effective radius r_{ef} of the aerosol mixture. Their conditional mathematical expectations are expressed by the regression equation $g_{XY} = \alpha_{XY} + \beta_{XY} y$, where $X = \{C_D, r_{ef}\}$, $Y = \{X_L(L_2)\}$, $\alpha_{XY} = 0.01-0.08$, β_{XY}

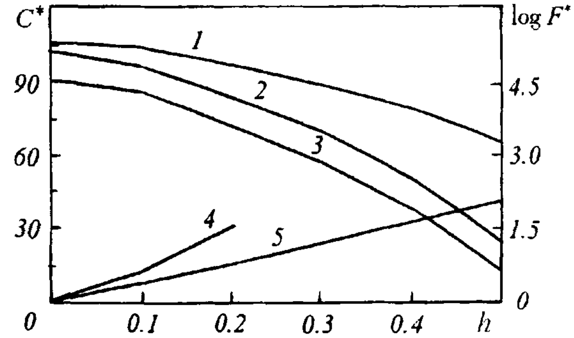


Fig. 3. Effect of the height h of the local relief on the fluxes F^* (curves 1-3) and the parameter C^* (curves 4 and 5) of scattered radiation. C^*, %; h, km.

= 0.01–0.017, and y is a fixed value of Y. The obtained value of C_D can be correlated with one or another stage of aerosol transformation. The spectral data also indicate that the expansion factors for the harmonics of order L_1 and L_2 will have the greatest weight in solving the transfer equation.

The reflective properties of an ocean surface experiencing a wind wave depend substantially on the wind speed and direction, the presence of internal waves and flows, and the degree of contamination of the aqueous medium. Waves play a decisive role in energy transfer between the atmosphere and the ocean. In modeling, an undulating surface is represented by a random function of elevations and slopes. Thus, the probability of the appearance of elementary areas of a water surface that have a normal with angular coordinates ϑ, φ is expressed by a distribution density $f(\vartheta, \varphi)$ that is a function of the wind direction and speed. Calculations employed the Cox–Munk distribution function [8]. For an undulating ocean surface, the coefficient of radiance β_r is introduced according to the equation

$$\beta_r = \frac{\pi}{\eta} f(\vartheta, \varphi)\, r(\vartheta', \varphi', \vartheta, \varphi), \tag{6}$$

where $r(\vartheta', \varphi', \vartheta, \varphi)$ is the reflection indicatrix expressed in terms of Fresnel coefficients. Foam is represented by an indicatrix corresponding to Lambert reflection and is accounted for by a weighting factor that depends on its relative area on the undulating surface.

To describe mathematically the local relief it is necessary to bring out the spatial structure of the unevennesses, their orientation relative to the radiation source and the receiver, the vertical dimensions of the relief unevennesses, and their gradients. An elementary area of the slope is introduced into the relief model by means of the optical characteristics of the reflection, determined by the spectral albedo and the light reflection indicatrix. Mathematical models of the relief were developed for the determinate and stochastic cases. A regular model of the relief was constructed on the basis of a ruled surface unbounded along the horizontal coordinates with a straight line parallel to the horizontal plane being its generatrix and a regular polygonal line symmetric with respect to this plane being its directrix. This model reflects the structure of a fairly wide class of actual relief shapes. A regular model can describe the micro- and mesoreliefs of plowlands, steppes, broken plains, semideserts, and deserts. In the limiting case where the slope height is $h = 0$, the model corresponds to a "smooth" surface.

Figure 3 illustrates the effect of a ridge-like relief on the fluxes of optical radiation. The relief is characterized by a contour interval of the slope $d = 1$ km and a height h varying from zero to 0.5 km. The atmosphere is cloudless with an optical thickness of 0.3 and a meteorological visibility $S_M = 20$ km. The aerosol scattering indicatrix corresponds to a model of continental mist. The light wavelength is 0.55 μm. The aperture of the receiver is 20°, the orientation in the vertical plane is $\vartheta = 10°$ (1), 0° (2, 4), $-5°$ (3), and 30° (5), and in the horizontal plane it is normal to the surface breaks. Calculated results show that an increase in the relief height leads to a marked decrease in the density of radiation fluxes F^* due to the screening effect. The shadow effect relative to the flux $F^*(h = 0)$ on a flatland is evaluated by the parameter $C^*(h) = [F^*(h = 0) - F^*(h)]/F^*(h = 0)$. Clearly, the ratio $C^*(h)$ increases monotonically with the height of the slopes.

A stochastic model of the relief was set up using a random height function $z(x, y)$, where x and y are the coordinates of a point in the horizontal plane. The distribution of surface heights conforms with to a normal law

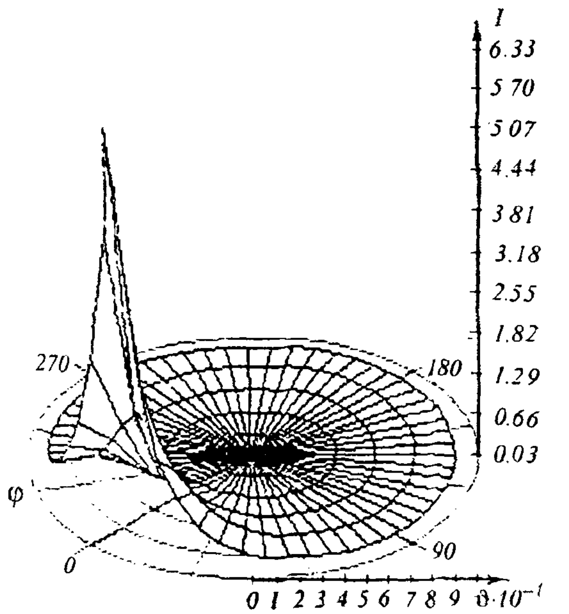


Fig. 4. Angular distribution of the intensity I (in relative units) of downward scattered radiation in a cloudless atmosphere at $S_M = 30$ km and $\varphi_s = 0^o$.

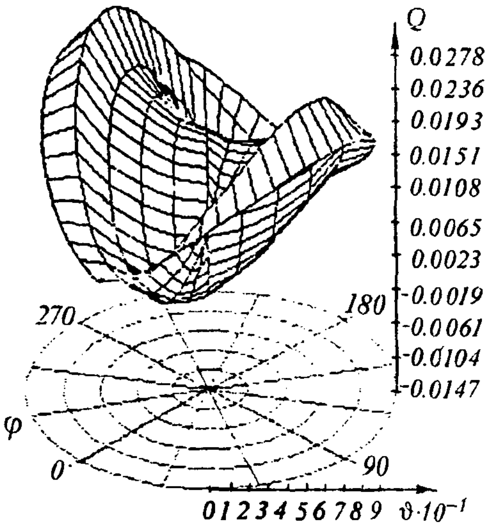
with a specified dispersion $D_z = \langle z^2(x, y)\rangle$, and the distribution of the inclination angles α between the normal to the surface and the vertical axis 0Z is taken to be uniform with a dispersion $D_\alpha = \langle \alpha^2(x, y)\rangle$. The mathematical expectations are $M[z(x, y)] = 0$ and $M[\alpha(x, y)] = 0$. The Fourier spectrum $W(p, q)$ of the random function of the relief was chosen from considerations of closer correspondence of the spectral characteristics of the modeled surface to the actual ones.

The anisotropic effect of light scattering by an aerosol found reflection in elaboration of the small-angle modification of the method of spherical harmonics of the theory of radiation transfer. Using the method of spherical harmonics in the small-angle approximation, we obtained a numerical solution for the angular distribution of the intensity $I(\vartheta, \varphi)$ of scattered radiation in a spherical terrestrial atmosphere.

Figure 4 illustrates an example of the obtained intensity distribution simultaneously over polar angles $\vartheta = 0-90^o$ and azimuthal angles $\varphi = 0-360^o$ for the hemisphere of directions corresponding to downward radiation in a cloudless atmosphere. Here, the origin of the cylindrical system of coordinates corresponds to the direction of sight at the zenith, and the radius corresponds to the polar angle of sight (in degrees) ϑ measured along a horizontal axis. The azimuth φ of the direction of sight (in degrees) corresponds to values measured on a scale of angles along the perimeter of an ellipse. The intensity of scattered radiation $I(\vartheta, \varphi)$ for the direction of sight specified by the angles ϑ and φ is presented on a vertical axis. As is seen from Fig. 4, under conditions of a cloudless atmosphere scattered radiation has maximum intensity in the direction ($\vartheta = \psi$, $\varphi = 0^o$) corresponding to the angular position of the sun. With cumulus cloudiness, the angular intensity distribution can have several maxima. This form of representing the calculated results facilitates analysis of the response of the characteristics of the field of scattered radiation to variations in the spatial distribution of the aerosol in the atmosphere.

A program for calculating the components I, Q, U, and V of the Stokes vector of scattered radiation in a spherical terrestrial atmosphere permits obtaining their angular distribution at an arbitrary height above the earth's surface for the hemispheres of directions corresponding to upward and downward radiation. Figure 5 illustrates the distribution of the second component Q of downward scattered radiation over the angles ϑ and φ. The angular distribution of the third component U of the Stokes vector also has a characteristic form. The fourth component V is negligible.

Calculated results for the dimensional components I, Q, U, and V permit determination of the following dimensionless parameters the degree P and azimuth χ of polarization, and the ellipticity β of scattered solar radiation:



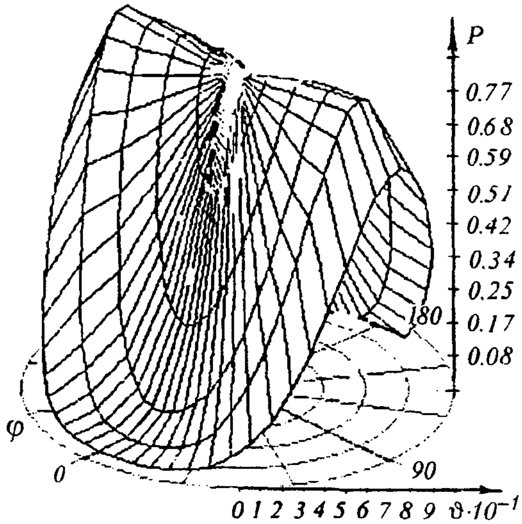


Fig. 5. Angular distribution of the Q component (in relative units) in a cloudless atmosphere for the hemisphere of directions corresponding to downward scattered radiation. S_M = 30 km. Sun at the horizon with the azimuth $\varphi_s = 0$.

Fig. 6. Distribution of the degree of polarization P of downward scattered radiation in a cloudless atmosphere over the polar ϑ and azimuthal φ angles. The remaining parameters are the same as in Fig. 5.

$$P = \frac{(Q^2 + U^2 + V^2)^{1/2}}{I}, \tag{7}$$

$$\chi = \frac{1}{2} \arctan \frac{U}{Q}, \tag{8}$$

$$\beta = \frac{1}{2} \arcsin \frac{V}{PI}. \tag{9}$$

Figure 6 presents the obtained distribution of the degree of polarization $P(\vartheta, \varphi)$. It has a characteristic form with a maximum in a zenith zone extended perpendicular to the plane of the sun's vertical. The angular distribution of the degree of polarization and its amplitude are indicators of atmospheric turbidity and the phase state of the suspended aerosol particles.

On the basis of data on the angular distribution of the scattered-radiation intensity $I(r, \psi, \vartheta, \varphi)$, integral characteristics are calculated, namely, the densities of fluxes of upward $F^{\uparrow}$ and downward $F^{\downarrow}$ radiation [10]. The difference of flux densities at a given level r

$$F(r, \psi) = F^{\uparrow}(r, \psi) - F^{\downarrow}(r, \psi) \tag{10}$$

yields the magnitude of the effective flux $F(r, \psi)$ (or of the shortwave radiation balance $B_s(r, \psi) = -F(r, \psi)$). Obviously, $F(r, \psi)$ is the projection of the light vector $\mathbf{F}(r\,\psi)$ in the vertical direction. The divergence of the field of the light vector div $\mathbf{F}$, taken with the opposite sign, expresses the radiation influx to an elementary volume of the medium, resulting in its radiation heating. The rate of radiation heating is calculated by the equation

$$\frac{\partial T}{\partial t} = -\frac{1}{c_p \rho} \operatorname{div} \mathbf{F}, \tag{11}$$

The total heating due to absorption of solar radiation is expressed as

$$\frac{\partial T}{\partial t} = \sum_{i=1}^{N} \frac{\partial T_i}{\partial t}, \tag{12}$$

where N is the number of intervals into which the solar spectrum is split in calculations and $\partial T_i/\partial t$ is the heating rate referred to the i-th spectral interval. For short time intervals $t-t_0$, the temperature at the level r is expressed by the linear function

$$T(r, t) = T(r, t_0) + \frac{\partial T(r, t)}{\partial t}(t - t_0), \tag{13}$$

where $T(r, t_0)$ is the temperature at the fixed instant of time t_0.

Radiation heating of a medium as a result of absorption of solar radiation is attenuated by the opposed process of radiation cooling in the longwave spectral range. There are a number of practical difficulties in the direct use of absorption spectra for calculating rates of radiation cooling. The main one of them lies in the high variability of the absorption index with a change in the wavelength in infrared vibrational spectra and the unfeasibility of accurate "line-by-line" summation. Attenuation within the limits of finite spectral bands belonging to radioactive gases is taken into account by introducing effective transmission functions $T_{\Delta\lambda}$. To calculate $T_{\Delta\lambda}$ use is made of data on the height profiles of meteorological parameters and the concentration of the main absorbing components, namely, water vapor, carbon dioxide gas, and ozone [6, 10, 11].

Conclusions. We developed a model of radiative transfer that is based on rigorous methods of calculating solar radiation on the basis of solving the transfer equation in the spherical system of atmosphere–earth's surface. We substantiated the possibility of adequately modeling the optical stratification of the air medium and the aqueous medium and of taking into account the effect of vapor-gas and aerosol layers of the wind wave of the water surface and the local relief on the radiation characteristics under conditions of multiple scattering.

The devised procedure for calculating the intensity and polarization characteristics of scattered solar radiation is adapted to numerical experiments for remote optical monitoring of natural and anthropogenic disturbances of the environment.

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NOTATION

$x(\theta)$, light scattering indicatrix; $x_m(\theta)$, $x_a(\theta)$ and $q_m(r)$, $q_a(r)$, light scattering indicatrices and weighting functions for the molecular and aerosol components of the air medium; θ, scattering angle; θ_0, scattering angle of direct rays; $\|x_{ij}(\theta)\|$, light scattering matrix; x_L^{ij}, coefficients of expansion of the elements of the light scattering matrix in generalized spherical functions; $P_L^{mn}(\mu)$, generalized spherical functions; μ, cosine of the scattering angle; ε, σ, α, indices of attenuation, scattering, and absorption of light, respectively; Λ, probability of survival of a light quantum; $k = \{D, W, S, O\}$, components of the surface aerosol; C_k, relative volume concentration of the components; L_k, number of the expansion coefficient corresponding to the k-th maximum; r_{ef} and r_{ef}^k, effective radii of particles of the aerosol mixture and its fractions; $I(r, \psi, \vartheta, \varphi)$, intensity of scattered light; r, distance from the earth's center to the observation point; ψ, polar angle of the sun; ϑ, polar angle of the line of sight; φ, azimuthal angle of the line of sight; φ_s, sun's azimuth; $\tau(r, \psi)$, optical length of the direct ray; πF_0, solar constant; $B(r, \psi)$, source function; S_M, meteorological visibility; P, degree of polarization of radiation; χ, azimuth of polarization of radiation; β, ellipticity parameter; $I_L^m(\mathrm{r})$, coefficients of series expansion of the intensity in spherical functions; $Y_L^m(\mathrm{s})$, spherical functions; r, radius vector of the observation point; s, unit vector of the direction of sight; $f(\vartheta, \varphi)$, distribution function of microareas of the undulating water surface over directions; $r(\vartheta', \varphi', \vartheta, \varphi)$, reflection indicatrix; β_r, coefficient of radiance; η, cosine of the angle of incidence; h, height of the slope; d, contour interval of the slope; D_z, D_α, dispersions of the elevations and slopes of the relief; $W(p, q)$, spectrum of the spatial frequencies p and q of the relief in the direction of the coordinate axes $0X$ and $0Y$; F^*, C^*, radiation flux and coefficient reflecting the effect of the relief; $F^{\uparrow}(r, \psi)$, upward radiation flux; $F^{\downarrow}(r, \psi)$, downward radiation flux; $F(r, \psi)$, effective flux;

$B_s(r, \psi)$, radiation balance; $\mathbf{F}(r, \psi)$, light vector; T, temperature; c_p, specific heat at constant pressure; ρ, density; t, time; t_0, fixed instant of time.

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